Modeling the Performance of Students Completing Technical Secondary Education in Rwanda: Quantile Regression Approach

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Abstract: **Quantile Regression Model (QRM) is preferred by current study than Ordinary least Square (OLS) because it does not only represent a central tendency of a distribution, but it is also robust to the presence of outliers as it does concern with the behavior at quantiles distribution. The objective of the study is to apply this approach to analyze and create robust models for data on the performance of 21595 and 21185 students who completed Technical Secondary Education in Rwanda for consecutive years 2013 and 2014 respectively. The study findings revealed that the option that a student pursued; and the knowledge of major technical courses, language, mathematics and practices have a positive effect on the performance of that student. Furthermore, the effect of language and mathematics on the performance is larger than that for other covariates. The nonparametric quantile regression methods that have been used to approve the results obtained by parametric quantile methods also exposed a positive effect of covariates on the response variable.**

Keywords: **Quantile, Regression, Performance, Covariates, Score.**

I. INTRODUCTION

1.1. Background of the study:

Quntile regression approach is recent and it was introduced by Koenker and Basset in 1978 who found that the unconditional quantile regression model is applied to many research areas especially in Econometric and Statistics. The theoretical and empirical studies on estimation of quantile regression models and models on performance of students have been conducted by various researchers and the most known literatures used by the study are Koenker (1978), Koenker and Xiao (2004), Xiao (2009), Yu, Lu and Stander (2003), Isabel, Javier, Vincente , Altea and Carmen (2003), Xiao(2005), Shan and Yang (2009), Basset and Koenker (2009), Ghouch and Genton (2010), Zongwu and Xiao(2009), Yen, Wang and Suen (2009), Eikner and Montondon (2001), Escanciano and Goh (2014), Zhijie (2005), Marcus, Harding and Lamarche (2011), Yeh, Wang and Suen (2009), Keming, Zudi and Stander(2003), Kim(2007), Lynn and Backmon (2006), Winter and Dodou (2011), and Koenker (2011).

The current study is focusing on modeling the performance of students completing Technical Secondary Education in Rwanda. That performance is defined by the overall grade that a student obtains as the award after passing national exams. To estimate the Quintile Regression Model, the study uses covariates like option that a student pursued, and knowledge of Mathematics, language, major technical courses and practices; and this knowledge is measured by the scores obtained from the courses done in national exams conducted in two consecutive years 2013 and 2014. The study uses different approaches for modeling by estimating the parametric and nonparametric quantile regression models. The study eventually evaluates the validity, accuracy and specifications of estimated models to make a comparison of results.

1.2. Statement of the problem:

The quantile regression model has an added advantage than the OLS model because it is robust to the preference of outliers and it doesn't only concern with the mean and median behavior (Yen, Wang and Suen, 2009). In addition Cody, Amy, Andrea, and Lawrence (2012) find that the quantile regression model produced estimates that were more unbiased than the estimates produced by linear regression model when the data do not follow the assumed distribution (Normality). Yu, Lu and Stander (2003) confirm that the standard linear regression model does not provide a complete picture of relationship between dependent and independent variables.

Based on findings of (Yen, Wang and Suen, 2009), Yu, Lu and Stander (2003) and Cody, Amy, Andrea, and Lawrence (2012) the models estimated using linear regression model are not robust, they need to be improved so that their estimates can be more unbiased and less sensitive to outliers as the linear regression model concerns mean and therefore mean is sensitive to outliers. Furthermore, there is no robust model estimated by the ministry of education in Rwanda or any other independent researcher to explain the model of student's performance in Rwanda.

1.3 Objectives and Hypothesis:

1.3.1 General objective

The general objective of the study is to apply quantile regression for modeling the performance of students completing technical secondary education in Rwanda using covariates such as option pursued by a student; and knowledge of mathematics, language, major technical courses and practices.

1.3.2 Specific objectives

The specific objectives of this study are as follows:

i. To investigate the applicability of quantile regression approach for Students performance in consecutive schools years 2013 and 2014.

ii. To model the effect of covariates (option pursued by a student and student's knowledge of Mathematics, Language, major technical courses and practices) on the performance of students.

iii. To fit the quantile regression model for the performance of students completing Technical Secondary Education in Rwanda given the highlighted covariates.

1.4 Research Hypotheses:

The research hypotheses of our study are as follows:

i. **H_0**: there is no difference between quantiles (equality of slopes) for Quantile Regression Models estimated for performance data in 2013 and 2014.

ii. **H_0:** there is no effect of covariates (option, major technical course, language, math and practice) on the performance of a student.

iii. **H_0:** $\beta_{i\tau} = 0$ (this means that the coefficients $\beta_{i\tau}$ in the fitted quantile regression model are not statistically significant).

II. METHODOLOGY

This research is based on quantitative research methods. It uses the secondary data collected and stored in examination database from Workforce Development Authority in Rwanda. The study firstly explores the data and investigates the distribution of the data in order to create models using the quantile functions. The study applies the theories and concepts developed in various literatures: Koenker (1978), Koenker and Machado (1999), Koenker and Xiao (2004), Basset and Koenker (2009) , Xiao(2005), Xiao (2009), Yu, Lu and Stander (2003), Isabel, Javier, Vincente , Altea and Carmen (2003), Shan and Yang (2009), Ghouch and Genton (2010) , etc.

2.1 Estimation of parametric Quantile regression model:

The standard linear regression model which is usually used to model the data is of the form

$$y = X^T \beta + \varepsilon \qquad (2.1)$$

Where $X = (1, x)^T$, $\beta = (\beta_0, \beta_1)^T$ y = dependent variable and x= independent variable.

This is OLS model which is has assumptions like normality, and it not robust as it only concerns with the conditional mean of distribution not quantiles of distributions. From this reason the study will focus on the theories of Quantile Regression Models (QRMs) and apply them to the performance of students completing Secondary Education in Rwanda.

Let Y be a random variable with cumulative distribution function $F_Y(y)$, we have

$$F_Y(y) = P(Y \le y) \tag{2.2}$$

The quantile function is given by

$$Q_Y(\tau) = F_Y^{-1}(\tau) = \inf\{y : F_Y(y) \ge \tau\},\ \tau \in [0,1] \tag{2.3}$$

From the estimated OLS model in (2.1) let deduce the parametric quantile regression model which is described as

$$y_i = \beta_{0\tau} + \beta_{1\tau} x_i + \varepsilon_{i\tau},\quad \forall i \in \{1, ..., n\} \tag{2.4}$$

Where $\beta_{0\tau}, \beta_{1\tau} \in R$ and $\varepsilon_{i\tau} \sim H_\tau$ verifying $H_\tau(0) = \tau$. The estimators of the model $\beta_{0\tau}$ *and* $\beta_{1\tau}$ are computed by solving the following equation

$$(\beta_{0\tau}, \beta_{1\tau}) = \underset{(\beta_{0\tau}, \beta_{1\tau}) \in R^2}{\arg\min} \{ \sum_{y_i > A} \tau |y_i - \beta_{0\tau} - \beta_{1\tau} x_i| + \sum_{y_i < A} (1-\tau) |y_i - \beta_{0\tau} - \beta_{1\tau} x_i| \} \tag{2.5}$$

The Quantile Regression Model (QRM) coefficients can be expressed in the general form

$$Q_\tau(y_i / x_i) = \sum_{j=1}^{p} \beta_{j\tau} x_{ij} \Leftrightarrow y_i = \sum_{j=1}^{p} \beta_{j\tau} x_{ij} + \varepsilon_{i\tau} \tag{2.6}$$

The assumption here is that $\varepsilon_{i\tau}$ has a distribution whose τ th quantile is Zero and it is Independent and Identically Distributed.

As OLS, the QRM use some measures like Standard Errors ($S_{\beta_{j\tau}}$) used to calculate confidence intervals (CI) and test hypothesis; in this case $C.I = \hat{\beta}_{j\tau} \pm Z_{\alpha/2} S_{\beta_{j\tau}}$

2.2 Tests for Quantile regression Model:

The current study uses test statistics suggested by Koenker and Machado (1999), and Yu, Lu and Stander (2003)

Let the null Hypothesis be $H_0 : \beta_{j\tau} = 0$

Where H_0 is rejected when $\left| \frac{(\hat{\beta}_{j\tau} - 0)}{S_{\beta_{j\tau}}} \right| > z_{\alpha/2}$

Let τ be a quantile we have the objective function which is

$$V(b(\tau)) = \sum_{y_i > x_t b} \tau |y_t - x_t b| + \sum_{y_i > x_t b} (1-\tau) |y_t - x_t b| \} = \sum \rho(\varepsilon_t) \tag{2.7}$$

where $\sum \rho(\varepsilon_t) = \varepsilon_t (\tau - I(\varepsilon_t < 0))$. For instance the case of τ =0.5, the Objective function is

$V(b(0.5)) = \sum_{y_i > x_t b} 0.5 |y_t - x_t b| + \sum_{y_i > x_t b} (1-0.5) |y_t - x_t b| \} = \sum_{y_i > x_t b} |y_t - x_t b|$, and this is the median conditional regression. This Objective function is different from the one for OLS which instead computes the mean conditional regression. To test the reliability of the QRM the study also uses the Likelihood Ratio test.

Let consider first of all the asymmetric Laplacian density of the form

$$f_1(u) = \tau(1-\tau)\exp\{-\rho_\tau(u))\} \tag{2.8}$$

The log likelihood under the assumption that $u_i = y_i - x_i'\beta$ will be

$$l(\beta) = n\log(\tau(1-\tau)) - \sum \rho_\tau(y_i - x_i'\beta) \tag{2.9}$$

Let consider the model of conditional quantile function

$$Qy_i(\tau / x) = x_{i1}'\beta_1(\tau) + x_{i2}'\beta_2(\tau) \tag{2.10}$$

The estimates $\hat{\beta}(\tau)$ are defined as the minimizers of objective function

$\hat{V}(\tau) = \min_{b\in R^p} \sum \rho_\tau(y_i - x_i'b)$, and $\tilde{\beta}(\tau)$ the minimizer of

$$\tilde{V}(\tau) = \min_{b_1\in R^{p-q}} \sum \rho_\tau(y_i - x_i'b_1) \tag{2.11}$$

Koenker and Machado (1999) denote that $\hat{V}(\tau)$ is the objective function of the unrestricted model and $\tilde{V}(\tau)$ is the objective function of the restricted model.

Let $\lambda^2 = \tau(1-\tau)$, from (2.11) we have

$$l(\hat{\beta}(\tau)) = \log(\tau(1-\tau)) - \sum \rho_\tau(y_i - x_i'\beta)$$

$$l(\tilde{\beta}(\tau)) = \log(\tau(1-\tau)) - \sum \rho_\tau(y_i - x_i'\beta_1)$$

$$-2\log\lambda_n(\tau) \equiv 2(l(\hat{\beta}(\tau) - l(\tilde{\beta}(\tau))) = 2(\tilde{V}(\tau) - \hat{V}(\tau)) \tag{2.12}$$

From the standard Likelihood Ratio Test $-2\log\lambda_n(\tau)$ Koenker and Machado (1999) show that under H_0 the likelihood test statistic is

$$LR = \frac{2\{\tilde{V}(b(\tau)) - \hat{V}(b(\tau))\}}{\lambda^2\omega(\tau)}\ , \tag{2.13}$$

In this case $\omega(\tau) = \dfrac{1}{f(\tau)}$, where $f(\tau)$ is the height of the error density at the chosen quantile τ and the identity (2.13) is asymptotically to χ_q^2

2.3 Goodness of fit for Quantile regression Model:

To calculate the goodness of fit for the parametric quantile regression, the study uses the goodness-of-fit which is the Pseudo-R^2 suggested by (Koenker & Machado, 1999) and (Furno, 2011)

$$Pseudo-R^2 = 1 - \hat{V}(\tau) / \tilde{V}(\tau) \tag{2.14}$$

2.4 Nonparametric methods for quantile regression:

In this study the researcher will use the nonparametric conditional quantile. In this case the study will use the nonparametric quantile model as suggested by Isabel, Javier, Vincente, Altea and Carmen (2003). The nonparametric regression model is

$$yi = f_\tau(x_i) + \varepsilon_{i\tau} \qquad \forall i \in \{1,\ldots,n\} \tag{2.15}$$

Where f_τ is unknown smooth function and $\tau \in [0,1]$, this assumption here is that the τ-th quantile of the error term $\varepsilon_{i\tau}$ conditional on the covariate x is assumed to be zero $Q_\tau(\varepsilon_{i\tau}(x_i|x) = 0$. The estimation of the smooth faction is of the form

$$\hat{f}_\tau(x) = \sum_1^n \omega_{\lambda,\tau}(x_i)y_i \tag{2.16}$$

Where λ is a smoothing parameter and $\omega_{\lambda,\tau}$ is the function of weights (kernel type, spline, etc) proposed by Koenker (2004).

III. RESULTS AND DISCUSSION

3.1 Dataset Description:

This study has one dependent variable (performance), four explanatory variables (Major technical courses, Math, Language and Practices) which are continuous and one explanatory variable (Option) which is nominal.

Table 1: Variables description

Variable	Description
Option	Option that a student pursued
Technical	Knowledge of the major technical courses
Math	Knowledge of Math
Language	Knowledge of language
Practices	Knowledge of practices (technical and professional)

The performance of students is measured by the grade obtained by a student after evaluating his performance in courses performed in national Exams. The grades vary from 0 to 60. The explanatory variables which are continuous are measured by scores or marks obtained in highlighted courses out of 100.

3.2 Descriptive Statistics for response and continuous covariates:

This study is analyzing the performance of 21595 and 21185 students who sat for national exams completing technical secondary education in Rwanda for two consecutive school years 2013 and 2014 respectively.

Table 2: Descriptive statistics for performance data

Year			Performance	Major technical course	Language	Math	Practices
2013	Number of Observations	Valid	21595	21591	21586	21593	21594
		Missing	0	4	9	2	1
	Std. Deviation		9.303	15.914	13.4569	12.831	7.645
	Skewness		0.843	0.352	0.62205	1.218	-0.699
	Minimum		0	0	0	0	0
	Maximum		57	97	90	89	99
	Percentiles	1st	5	4	2	0	63
		5th	8	12	6	2	69
		10th	9	16	9	3	72
		25th	12	25	16	6	76
		50th	18	35	24	14	81
		75th	25	47	34	23	87
		90th	33	58	44	34	91
		95th	38	65	50	42	93
		99th	46	78	62	57	96

Year			Performance	Major technical course	Language	Math	Practices
2014	Number of Observations	Valid	21185	21098	21098	21099	21157
		Missing	0	87	87	86	28
	Std. Deviation		9.969	21.8	17.485	19.99	7.972
	Skewness		0.447	-0.14	0.097	0.621	-0.66
	Minimum		0	0	0	0	17
	Maximum		59	100	98	94	100
	Percentiles	1st	4	4	8	0	61
		5th	6	12	16	1	70
		10th	8	19	23	3	72
		25th	13	34	34	9	77
		50th	20	51	46	23	83
		75th	28	67	57	41	88
		90th	35	78	69	56	92
		95th	38	84	76	64	94
		99th	45	92	87	77	97

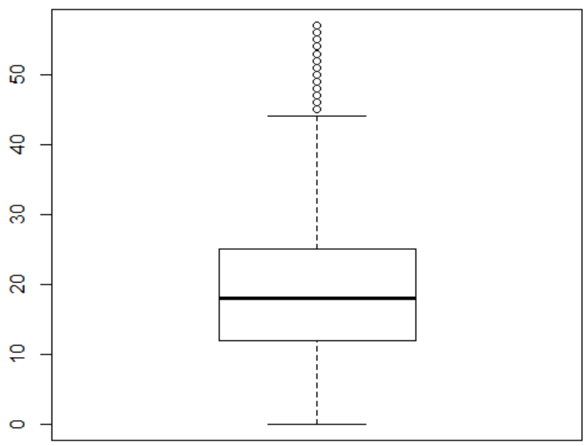
The data shows that the minimum grade is 0 in both two consecutive school years while the maximum grade is 57 in 2013 and 59 in 2014. Note that the optimal maximal grade should be 60. For the covariates the minimum score is 0 out of 100 in both two schools years except the minimal score for practices in 2014 which is 17. The maximum score vary per covariate and per school year.

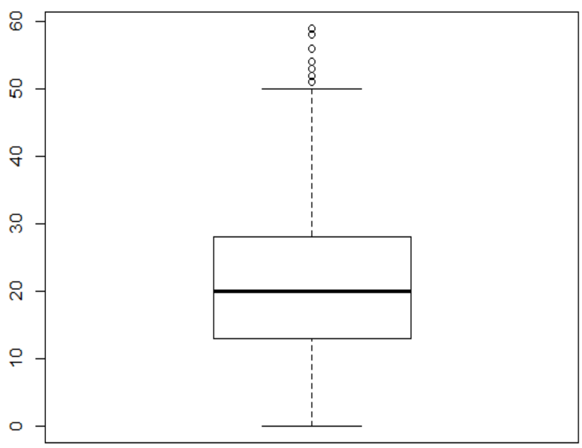
The skewness of the data shows that for some variables the frequency distributions are positively skewed others are negatively skewed, this means that the data are not perfectly normally distributed for all variable. This implies that the quantile regression approach would be suitable for the model. As the aim of this study is to estimate the quantile regression model, where the study should examine the estimates behaviors at each quantile of distribution (lower, medium and upper); the table 2 also shows the optimal grades (for performance) or optimal scores (for covariates) at each percentile (1st , 5th, 10th, 25th, 50th, 75th, 95th and 99th).

3.3 Detecting outliers in response variable and covariates:

To detect the outliers, the study is using the much known approach called boxplot. Before estimating the quantile regression model detecting outliers is essential as Yen, Wang and Suen, (2009) argue that the added value for quantile regression approach to OLS approach is that the first approach can also concern the influential values while the last approach cannot. The presence of outliers means that OLS regression would be biased and inaccurate when used because it can be highly influenced by those outliers.

a) Performance:





b) Covariates:

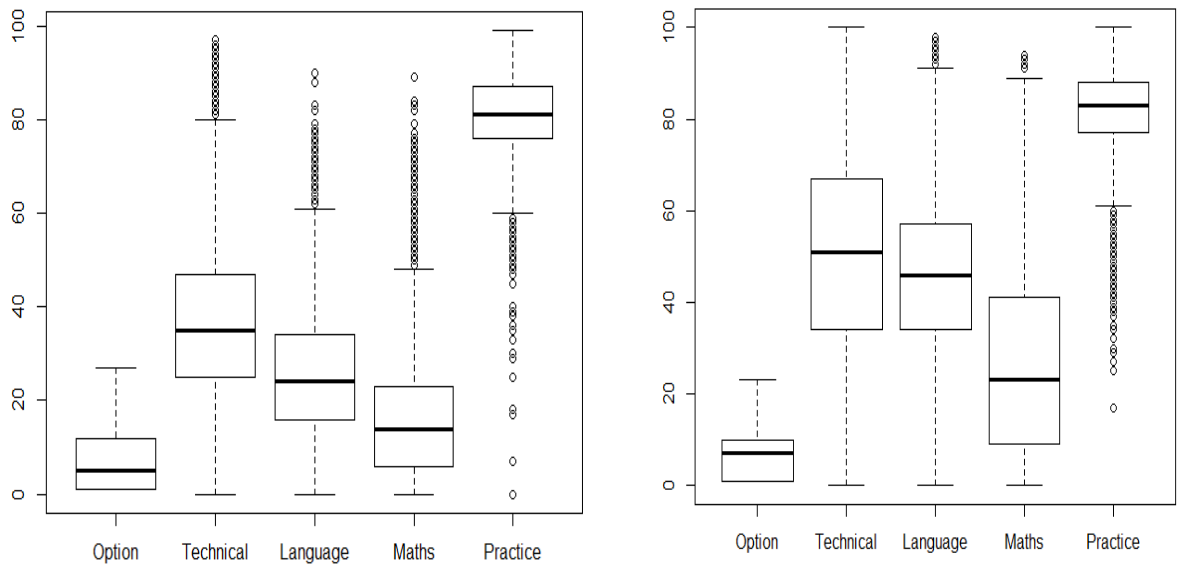


Figure 1: Boxplots of response variable and covariates in 2013 and 2014

The boxplots of performances show the outliers in the data for 2013 and 2014. Otherwise, covariates have outliers except the option that a student pursued in both consecutive school years and technical major courses in 2014 (fig.1). This implies that the Quantile regression approach would be preferred to create models than the OLS model.

3.4 Normality Test:

Cody, Amy, Andrea, and Lawrence (2012) find that both OLS and Quantile regression methods performed well in the simulations when the data follow the assumed distribution; but when the data do not follow the assumed distribution the least square estimates are often biased. In the current study the quantile regression model doesn't assume any distribution; to sow that it is better performing than the ignored OLS model let first test the normality. To test the normality for the observation greater than 5000 with Shapiro-Wilk W test do not apply, from this reason the study used the Kolmogorov-Smirnov test in order to test the normality of the performance for students completing Secondary schools in Rwanda as the number of observations is greater than 5000.

Table 3: Kolmogorov-Smirnov test

School year	Statistic	P-Value
2013	D = 0.0060107	p-value = 0.4162
2014	D = 0.0053352	p-value = 0.5827

Alternative hypothesis: two-sided

From table 5 the study failed to reject the null hypothesis that the data is normally distributed at significance level of 5%. This implies that the regression model used by the current study does not assume any distribution; therefore OLS methods are not suitable to estimate the model.

3.5 Quantile regression model:

3.5.1 Test of equality of slopes:

The test in table 4 is known as Wald test and it is applied for the performance data for 2013 and 2014 at the quantiles (0.1, 0.2, 0.25, 0.5, 0.75 and 0.95). H_0: the estimates for quantiles are statistically equal versus H1: the estimates for all quantiles are not statistically equal.

Table 4: Test of equality of slope for performance data

School year	Df	Resid	Df	F value	Pr(>F)
2013	1	25	129473	120.36	2.2e-16 ***
2014	1	25	126467	86.919	2.2e-16 ***

*Signif. codes: .000 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1*

Notes: Df = degree of freedom, Resid= residuals, F Value= Fisher Statistic, Pr = P- Value

Considering the significance column of table 4, the information reveals that the P-value is less than .000; therefore the test significantly rejects equality of the estimated coefficients for the quantiles in each case. This also implies that the use of quantile regression approach is appropriate in the research.

3.5.2 Model Construction and Interpretation:

In the model there are coefficients estimate for selected sample quantiles:10^{th} , 25^{th}, 50^{th} , 75^{th} and 95^{th} ; the standards errors and confidence interval for quantile regression coefficient estimates which are obtained by bootstrapping methods suggested by Koenker and Hallock (2001).

Table 5: Coefficients for quantile regression model for performance in 2013

Performance	Coef.	Std. Err.	t	P>t	[95% Conf.Interval]	
q10						
Option	0.0921183	0.0051545	17.87	0.000	0.0820151	0.102222
Major technical Courses	0.1630191	0.0024063	67.75	0.000	0.1583025	0.167736
Language	0.1950206	0.0058977	33.07	0.000	0.1834606	0.206581
Mathematics	0.2092207	0.0036255	57.71	0.000	0.2021144	0.216327
Practices	0.1153451	0.0055862	20.65	0.000	0.1043958	0.126295
_cons	-9.764805	0.4417941	-22.1	0.000	-10.63075	-8.89886
q25						
Option	0.1274816	0.0072566	17.57	0.000	0.1132581	0.141705
Major technical Courses	0.1877471	0.0016519	113.65	0.000	0.1845092	0.190985
Language	0.2200695	0.0041505	53.02	0.000	0.2119342	0.228205
Mathematics	0.2080796	0.0046202	45.04	0.000	0.1990236	0.217136
Practices	0.162192	0.0036818	44.05	0.000	0.1549753	0.169409
_cons	-13.26633	0.3136032	-42.3	0.000	-13.88102	-12.6516
q50						
Option	0.2037095	0.0062494	32.6	0.000	0.1914603	0.215959
Major technical Courses	0.2172196	0.0015307	141.91	0.000	0.2142192	0.22022
Language	0.2336521	0.0034722	67.29	0.000	0.2268464	0.240458
Mathematics	0.2035601	0.002727	74.65	0.000	0.1982149	0.208905
Practices	0.2059704	0.0047916	42.99	0.000	0.1965785	0.215362
_cons	-15.87299	0.4238257	-37.45	0.000	-16.70372	-15.0423
q75						
Option	0.2536422	0.0112317	22.58	0.000	0.2316272	0.275657
Major technical Courses	0.2412297	0.0039738	60.7	0.000	0.2334407	0.249019
Language	0.2407663	0.003659	65.8	0.000	0.2335945	0.247938
Mathematics	0.1955447	0.0032878	59.48	0.000	0.1891004	0.201989
Practices	0.2233014	0.0052867	42.24	0.000	0.212939	0.233664
_cons	-15.62864	0.4220181	-37.03	0.000	-16.45583	-14.8015
q95						
Option	0.3185911	0.0122063	26.1	0.000	0.2946659	0.342516
Major technical Courses	0.2786875	0.0043599	63.92	0.000	0.2701417	0.287233
Language	0.1914168	0.0050296	38.06	0.000	0.1815584	0.201275
Mathematics	0.2336471	0.0056235	41.55	0.000	0.2226246	0.24467
Practices	0.2368048	0.0066949	35.37	0.000	0.2236822	0.249927
_cons	-13.83869	0.5153053	-26.86	0.000	-14.84873	-12.8287

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Table 6: Coefficients for quantile regression model for performance in 2014

Performance	Coef.	Std. Err.	t	P>t	[95% Conf.Interval]	
q10						
Option	0.0199507	0.0061567	3.24	0.001	0.0078832	0.032018
Major technical courses	0.1581472	0.0030692	51.53	0.000	0.1521313	0.164163
Language	0.1843354	0.0048363	38.11	0.000	0.1748558	0.193815
Mathematics	0.1332119	0.0031845	41.83	0.000	0.12697	0.139454
Practices	0.0524135	0.0053582	9.78	0.000	0.041911	0.062916
_cons	-9.043629	0.5055605	-17.89	0.000	-10.03457	-8.05269
q25						
Option	0.0602142	0.0074582	8.07	0.000	0.0455954	0.074833
Major technical courses	0.1846391	0.0032509	56.8	0.000	0.1782671	0.191011
Language	0.1837471	0.0024248	75.78	0.000	0.1789942	0.1885
Mathematics	0.1421237	0.0026636	53.36	0.000	0.1369028	0.147345
Practices	0.0648927	0.0036818	13.29	0.000	0.055319	0.074467
_cons	-9.449224	0.3850155	-24.54	0.000	-10.20388	-8.69456
q50						
Option	0.1326493	0.0111466	11.9	0.000	0.1108011	0.154498
Major technical courses	0.2036608	0.0021983	92.65	0.000	0.199352	0.20797
Language	0.1833922	0.0022108	82.95	0.000	0.1790588	0.187726
Mathematics	0.1464316	0.0025477	57.48	0.000	0.141438	0.151425
Practices	0.0904645	0.0057529	15.72	0.000	0.0791883	0.101741
_cons	-10.09226	0.4546169	-22.2	0.000	-10.98335	-9.20118
q75						
Option	0.177331	0.0094713	18.72	0.000	0.1587665	0.195895
Major technical courses	0.2147156	0.0023766	90.34	0.000	0.2100572	0.219374
Language	0.1758496	0.0018808	93.5	0.000	0.1721631	0.179536
Mathematics	0.146898	0.0026854	54.7	0.000	0.1416345	0.152162
Practices	0.1324806	0.0049829	26.59	0.000	0.1227138	0.142248
_cons	-11.23974	0.4067193	-27.64	0.000	-12.03694	-10.4425
q95						
Option	0.2156958	0.0183219	11.77	0.000	0.1797834	0.251608
Major technical courses	0.2169203	0.0031085	69.78	0.000	0.2108273	0.223013
Language	0.1793492	0.0054715	32.78	0.000	0.1686247	0.190074
Mathematics	0.1440419	0.0033224	43.35	0.000	0.1375297	0.150554
Practices	0.1899956	0.0053046	35.82	0.000	0.1795983	0.200393
_cons	-12.29942	0.4432366	-27.75	0.000	-13.16819	-11.4306

To test the coefficients from tables 5and 6, the study is using test statistics suggested by Koenker and Machado (1999), and Yu, Lu and Stander (2003). The null hypothesis is

$$H_0 : \beta_{j\tau} = 0 ,$$

where $\beta_{j\tau}$ are coefficients of covariates: option, major technical course, language, mathematics and practices at quantiles τ . This model is addressing the alternative hypothesis that the coefficient $\beta_{j\tau}$ in the fitted quantile regression models from the performance data of 2013 and 2014 is statistically significant at all quantiles (q10, q25, q50, q75 and q95).

3.5.3 Goodness of fit for quantile regression model:

As the R^2 is used to calculate the goodness of fit for OLS model, the quantile regression model uses the pseudo R^2 and it is estimated across the quantiles.

Table 7: R squared estimates for multiple quantile regression analysis of performance data

Quantile	**Pseudo R squared in 2013**	**Pseudo R squared in 2014**
0.1	0.4856	0.5551
0.25	0.5184	0.5223
0.5	0.5599	0.5873
0.75	0.5943	0.6073
0.95	0.612	0.6025

The analysis shows that the Pseudo-R^2 varies quantile by quantile; it is general small to the lower tails of distribution than the higher quantiles of distribution. The Pseudo-R^2 of the current case means that these five predictors (Option, Major Technical course, language, math and practice) account for between 48.6% and 61.2% in 2013 and between 55.5% and 60.2% in 2014 of the variance at different levels of performance. Therefore, the evaluation of the model implies that it is not weak at all quantiles.

3.5.4 Plot the quantile regression:

Most of the models on the performance of students used conventional OLS approach. However, based on the findings of Yen, Wang and Suen (2009) the OLS methods result the model with estimates at conditional mean and these estimates would be unbiased if and only if the distribution is not tailed or when the distribution of data has no outliers. Each panel in fig. 2 and fig.3 plots one coordinate of the parameter vector β_τ as a function of τ , $\tau \in [0,1]$, in the plot the study did not take all values between 0 and 1, the study takes the values from 0.05 to 0.95, and it examines the behavior of estimates on different quantiles in that interval. For each covariate, those point estimates are interpreted as the effect of one unit change of the covariate on the performance of a student holding other covariates fixed.

The plot has two scales: the horizontal scale indicates the axis of quantile and the vertical scale indicate the effect of the covariate on the performance of the students. The red solid line in each box in the figure shows the OLS estimates of the conditional mean effect. The two dotted red lines represent 95 percent confidence interval for the OLS estimates. The figures show that the confidence interval for OLS estimate in each covariate does not change for all quantiles, and this means that the effect of the covariates' coefficients are constant across the conditional distribution of the independent variables. However the effect of the covariates 'coefficients for quantile regression model is not constant across the conditional distribution.

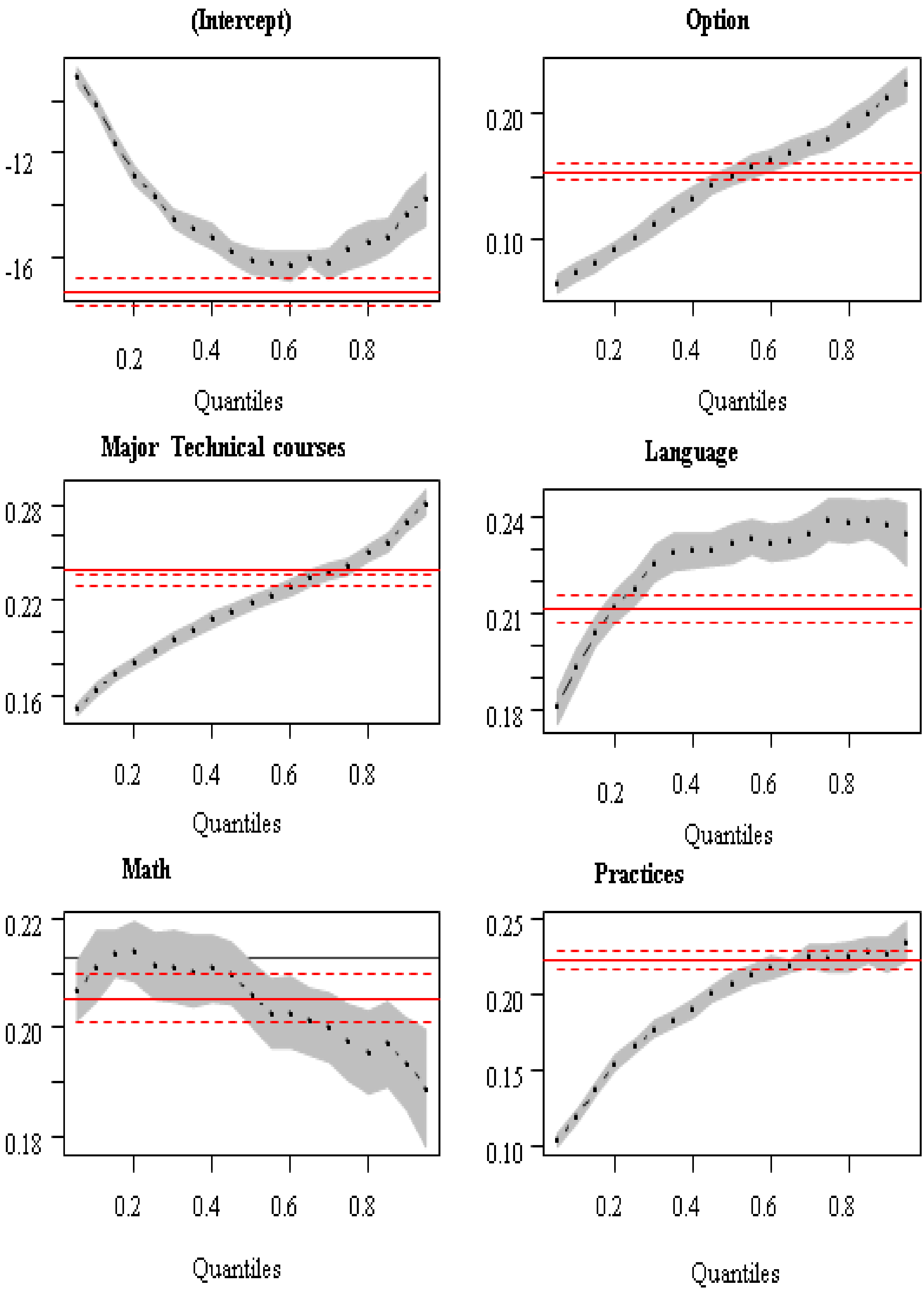

Figure 2: Graphical representation of the quantile regression estimates for the performance in 2013

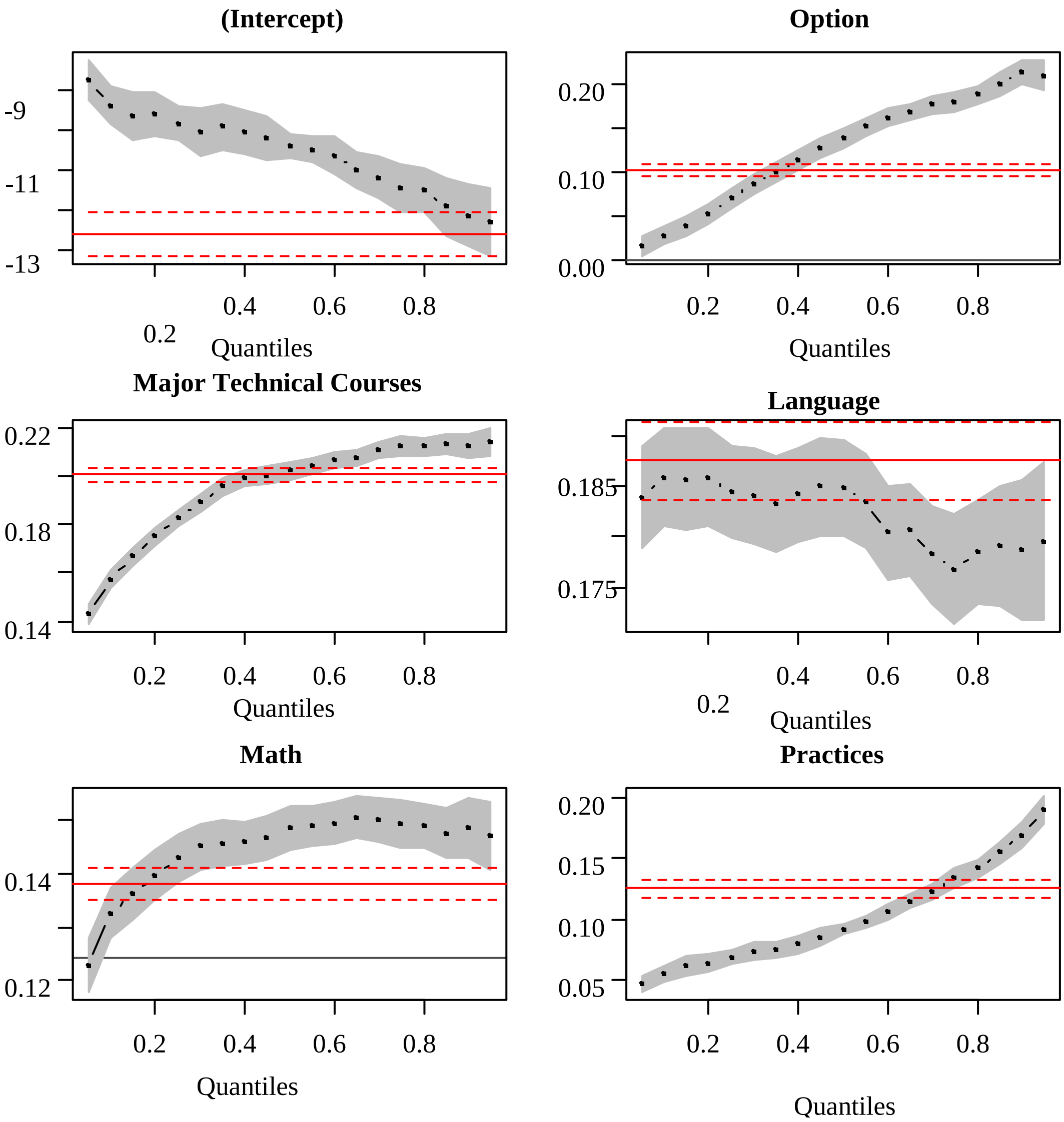

Figure 3: Graphical representation of the quantile regression estimates for the performance in 2014

Fig. 2 and fig. 3 show that the effect of option; and knowledge of language, major technical courses, math and practice is significantly positive in two sequential years 2013 and 2014.The effect of the option that a student pursued on his performance is almost the same for lower tails of distribution as higher quantiles of distributions. The effect of major technical courses on the performance is small at lower quantiles than higher tails of distribution in 2014, but the effect is almost the same for all quantiles in 2013. The effect of Language in 2013 increases gradually as the levels of quantile increases, this means that that effect is weak at lower tails of distribution than the higher quantile of distributions. The data of 2014 indicates that the language has a very large positive effect on the performance of a student compared the effect of the language on performance of student in 2013; this is shown by the large confidence interval, where very large positive effect is observable at lower tails and at the higher quantiles of distribution. The effect of math on the performance is larger in 2013 especially at lower tails of distribution and upper quantiles of distribution. In the data of 2014, the effect of math on the performance of the student is also large but it is small at lower quantiles of distribution and increases as the level of quantiles increases. The effect of practical skills is small at lower quantiles of distribution and great at upper quantiles in 2013, while the effect is almost the same for all quantiles of distribution in 2014.

3.6 Non parametric quantile regression Approach:

The current study predict the performance of students completing Technical Secondary Education using the nonparametric quantile regression smoothing spline model suggested by Koenker (2011) and MCMillen (2013) to improve the flexibility of the model and to have more accurate prediction. This is the additive model for non-parametric quantile regression using total variation penalty method. The solid curves are conditional quantile functions (Smooth function with smoothing parameter $\lambda = 2$, and increasing function without smoothing parameter).

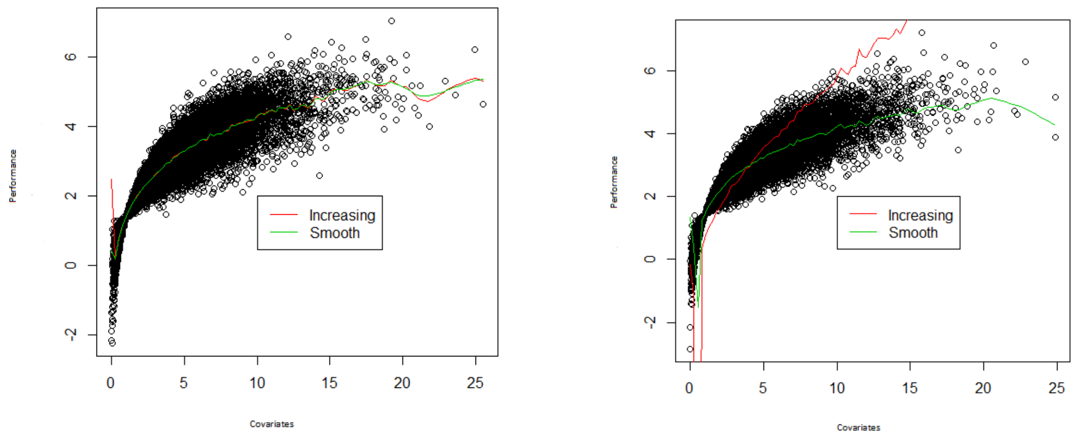

Figure 4: Predicted median Responses at mean value of Z for performance data in 2013 and 2014

Fig. 4 illustrates the relationship between the covariates (option, major technical course, language, math and practice) and the performance in two consecutive years 2013 and 2014; the curve is an estimate of 0.50 conditional quantile functions subject to the total variation of the function gradient. Z is defined as probability density function of normal distribution. The curves show that the relationship between covariates and the performance is positive; this is confirming the results obtained using parametric quantile regression. However, in the 2013 the increasing curve and smoothing curve are almost the same but in 2014 those two curves are different.

IV. CONCLUSION AND RECOMMENDATIONS

The current study is aimed at applying the quantile regression model in the performance of the students who completed the Technical Secondary Education in Rwanda for two consecutive school years of 2013 and 2014. The study concludes that the quantile approach is applicable for the performance data in 2013 and 2014 and it is more suitable than OLS approach as the data represent some outliers and the covariates are not normally distributed for allowing the model to assume the normal distribution. The study also revealed that there is a positive effect of all covariates (Option, major technical course, language, math and practice) on the performance of students. However, language and math showed larger positive effect on the performance of students than other covariates. The study also used the nonparametric method in order to have more accurate prediction and to improve the flexibility of the model which was previously estimated with parametric methods; and this method confirmed the findings from parametric estimates.

Based on the finding from this study, the recommendations are formulated in order to contribute to the improvement of models created by studies in various areas in general and the performance of students in particular. The quantile regression approach must be used in different areas of research in Rwanda in order to minimize errors and increase the robustness. The students studying technical options and preparing the national exams must focus on all subjects especially major technical courses, language, math and practice but more emphasis must be put in increasing their mathematical and language skills as they contribute a lot on their performance. This study has only used some academic factors that contribute on the performance of student using quantile regression approach; a further study must be carried out to investigate the effect of nonacademic factors like gender of student, his location and his background using the similar approach.

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